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ON ARTIN COKERNEL OF GROUP ($Q_{2M}\times C_4$) WHEN M IS AN ODD NUMBER

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ABSTRACT

The main purpose of this paper is to find The Artin Cokernel of the group ($Q_{2m}\times C_4$), which is denoted by AC ($Q_{2m}\times C_4$) where Q_{2m} is the Quaternion group and C_4 is the cyclic group of order 4 . We have also found the general form of Artin's characters table of Ar($Q_{2m}\times C_4$) and the Rational characters table when m is an odd Number

Key word: *Quaternion group, the cyclic group, Artin's characters, Artin's characters table, the Rational characters table and Artin Cokernel .*

Mathematics subject Classification: *16G30, 17B10*

I. INTRODUCTION

The square matrix whose rows correspond to Artin's characters and columns correspond to the Γ-classes of G is called Artin's characters table . this matrix is very important to find the cyclic decomposition of the factor group AC(G) and Artin's exponent A(G). In 1967 T.Y. lam [9] studied A(G) extensively for many groups In 1981 C.Curits and I. Reiner[3] studied Methods of Representation Theory with Application to Finite Groups. In 2008 A.H. Abdul-Munem [1] studied the general from of Artin Cokernel of The Quaternion group Q_{2m} when m is an odd number.

The aim of this paper is to find the general from of The Artin Cokernel, the Artin's characters table and the Rational characters table of the group ($Q_{2m}\times C_4$) when m is an odd Number.

II. PRELIMINARIES

This section introduce some important definitions and basic concepts , The Artin characters table, The Rational characters table the factor group AC(G) of a group G and the matrix M(G), M(Q_{2m}),P(Q_{2m}) and W(Q_{2m}).

Theorem1:[4]Let T_1: $G_1\rightarrow GL(n,F)$ and T_2: $G_2\rightarrow GL(m,F)$ be two irreducible representations of the groups G_1 and G_2 with characters χ_1 and χ_2 respectively then :

$T_1\otimes T_2$ is irreducible representation of the group $G_1\times G_2$ with the character $\chi_1\cdot\chi_2$.

Theorem 2: [6] Let H be a cyclic subgroup of G and $h_1, h_2, \dots, h_m$ are chosen as representative for m-conjugate classes of H contained in CL(g) in G, then :

1- $\varphi'(g) = \dfrac{|C_G(g)|}{|C_H(g)|}\sum_{i=1}^{m}\varphi(h_i)$ if $h_i \in H \cap CL(g)$

2- $\varphi'(g) = 0$ if $H \cap CL(g) = \phi$.

Definition 1:[9] Let G be a finite group, all characters of G induced from a principal character of cyclic subgroups of G are called **Artin's characters of G.**

In theorem 2 , if φ is the principal character , then $\varphi(h_i) = \varphi(1) = 1$, where $h_i \in H$

Proposition 1:[3] The number of all distinct Artin's characters on a group G is equal to the number of Γ-classes on G. Furthermore, Artin's characters are constant on each Γ-classes.

Definition 2: [2] Artin's characters of finite group G can be displayed in a table **called Artin's characters table of G** which is denoted by Ar (G).

The first row is the Γ- conjugate classes, the second row is the number of elements in each conjugate classes, the third row is the size of the centralize $|C_G(CL_\alpha)|$ and the rest rows contain the values of Artin's characters.

Theorem 3: [1] The Artin's characters table of the Quaternion group Q_{2m} when m is an odd number is given as follows :

Table 1 Artin's characters table of the Quaternion group Q_{2m} when m is an odd number

Γ- classes	Γ- classes of C_{2m}								
	x^{2r}				x^{2r+1}				[y]
$\|CL_\alpha\|$	1	2	…	2	1	2	…	2	2m
$\|C_{Q_{2m}}(CL_\alpha)\|$	4m	2m	…	2m	4m	2m	…	2m	2
Φ_1	2Ar(C_{2m})								0
Φ_2									0
⋮									⋮
Φ_l									0
Φ_{l+1}	m	0	…	0	m	0	…	0	1

where $0 \leq r \geq m-1$, l is the number of Γ-classes of C_{2m} and Φ_j are the Artin characters of the quaternion group Q_{2m} , for all $1 \leq j \leq l+1$.

Definition 3:[7]A rational valued character θ of G is a character whose values are in Z, which is $\theta(g) \in Z$, for all $g \in G$.

Corollary 1:[8]The rational valued characters $\theta_i = \sum_{\sigma \in Gal(Q(\chi_i)/Q)} \sigma(\chi_i)$ form the basis for $\overline{R}$(G), where χ_i are the irreducible characters of G, and their numbers are equal to the number of conjugacy classes of cyclic subgroup of G .

Definition 4: [8] The complete information about rational valued characters of a finite group G is displayed in a table called **rational valued characters table of G**. We refer to it by $\overset{*}{\equiv}$(G) which is n×n matrix whose columns are Γ-classes and rows which are the values of all rational valued characters of G, where n is the number of Γ-classes.

Definition 5:[8] Let T(G) be the subgroup of $\overline{R}$(G) generated by Artin's characters .T (G) is normal subgroup of $\overline{R}$(G) and denotes the factor abelian group $\overline{R}$(G)/T(G) by AC(G) which is called **Artin cokernel of** G.

Definition 6:[7] Let M be a matrix with entries in a principal domain R. **A k-minor of M** is the determinant of k×k sub matrix preserving row and column order.

Definition 7:[7] A k-th determinant divisor of M is the greatest common divisor (g.c.d) of all the k-minors of M.This is denoted by D_k (M)

Lemma 1:[7] Let M, P and W be matrices with entries in a principal ideal domain R, let P and W be invertible matrices ,Then D_k (P M W)= D_k (M) module the group of unites of R.

Theorem 4:[7] Let M be an n × n matrix with entries in principal ideal domain R, then there exist two matrices P and W such that:

1. P and W are invertible.
2. P M W = D.
3. D is diagonal matrix.
4. if we denote D_{ii} by d_i then there exists a natural number m ; $0 \le m \le n$ such that $j > m$ implies $d^j = 0$ and $j \le m$ implies $d^j \ne 0$ and $1 \le j \le m$ implies $d^j | d^{j+1}$.

Definition 8:[7] Let M be matrix with entries in a principal domain R, be equivalent to a matrix D = diag $\{d_1, d_2, \dots, d_m, 0, 0, \dots, 0\}$ such that $d_j | d_{j+1}$ for $1 \le j < m$

We call D **the invariant factor matrix of** M and $d_1, d_2, \dots, d_m$ the invariant factors of M.

Theorem 5:[7] Let K be a finitely generated module over a principal domain R, then K is the direct sum of cyclic sub module with an annihilating ideal $<d_1>, <d_2>, \dots, <d_m>$, $d_j | d_{j+1}$ for j = 1, 2, … , K-1.

Proposition 2:[8] AC(G) is a finitely generated Z- module .Let m be the number of all distinct Γ-classes then Ar(G) and ≡*(G) are of the rank l. There exists an invertible matrix M(G) with entries in rational number such That: $\equiv^*(G)=M^{-1}(G).Ar(G)$ and this implies $M(G)=Ar(G).(\equiv^*(G))^{-1}$

Theorem 6:[5] $AC(G) = \bigoplus_{i=1}^{l} C_{d_i}$ where $d_i = \pm D_i(G) / D_{i-1}(G)$ where l is the number of all distinct Γ-classes.

Corollary 2:[8] $|AC(G)| = |\det(M(G))|$

Lemma 2:[8] If A and B are two matrices of degree m and t respectively, then:

$\det(A\otimes B) = (\det(A))^{t} \cdot (\det(B))^{m}$.

Lemma 3:[8] Let A and B be two non-singular matrices of rank l and m respectively, over a principal domain R and let:

$P_1AW_1 = D(A) = \mathrm{diag}\{d_1(A), d_2(A), \ldots, d_l(A)\}$ and $P_2AW_2 = D(B) = \mathrm{diag}\{d_1(B), d_2(B), \ldots, d_m(B)\}$The invariant factor matrices of A and B then:

$(P_1 \otimes P_2)(A \otimes B)(W_1 \otimes W_2) = D(A) \otimes D(B)$

and from this the invariant factor matrices of $A \otimes B$ can be obtained.

Proposition 3:[5] Let H_1 and H_2 be $p_1$and p_2 - groups respectively where p_1 and p_2 are distinct primes and if M_1 is the matrix from all cyclic subgroups of $\overline{R}(H_1)$ basis and M_2 is the matrix which expresses the $T(H_2)$ basis terms of $\overline{R}(H_2)$ basis then the matrix which expresses the $T(H_1 \times H_2)$ basis of $\overline{R}(H_1 \times H_2)$ basis is $M_1 \otimes M_2$.

Proposition 4:[1]If m= $p_1^{\alpha_1}.p_2^{\alpha_2}.......p_n^{\alpha_n}$ where g.c.d(p_i,p_j)=1 , if i≠j and p_i 's are primes numbers, and α_n any positive integers, then the matrix $M(Q_{2m})$ of the quaternion group Q_{2m} is :

$$M(Q_{2m})=\left[\begin{array}{ccccc|ccccc|c}
 & & & & 1 & & & & & 1 & 1\\
 & & & & 1 & & & & & 1 & 1\\
 & & 2R(C_m) & & \vdots & & & 2R(C_m) & & \vdots & \vdots\\
 & & & & 1 & & & & & 1 & 1\\
0 & 0 & \cdots & 0 & 1 & 0 & 0 & \cdots & 0 & 1 & 1\\
\hline
 & & & & 1 & 0 & 0 & \cdots & 0 & 1 & 0\\
 & & & & 1 & 0 & 0 & \cdots & \vdots & 1 & 0\\
 & & 2R(C_m) & & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots\\
 & & & & 1 & 0 & 0 & \cdots & 0 & 1 & 0\\
0 & 0 & \cdots & 0 & 1 & 0 & 0 & \cdots & 0 & 1 & 0\\
\hline
1 & 1 & \cdots & 1 & 1 & 0 & 0 & \cdots & 0 & 0 & 0
\end{array}\right]$$

Which is $[2(\alpha_1+1).(\alpha_2+1)\ldots.(\alpha_n+1)+1]\times[2(\alpha_1+1).(\alpha_2+1)\ldots.(\alpha_n+1)+1]$ square matrix, where $R(C_m)$ is the matrix which is obtained by omitting the last row {0,0,…,0,1}and the last column {1,1,…,1}from the tensor product, $M(C_{p^1}^{\alpha^1})\otimes M(C_{p^2}^{\alpha^2})\otimes ..M(C_{p^n}^{\alpha^n})$ where $M(C_m)$ is of order,$[(\alpha_1+1).(\alpha_2+1)\ldots.(\alpha_n+1)]\times[(\alpha_1+1).(\alpha_2+1)\ldots.(\alpha_n+1)]$ square matrix

Proposition 5:[1]If m is odd then the matrix $P(Q_{2m})$ and $W(Q_{2m})$ are taking the forms:

$$P(Q_{2m})=\left[\begin{array}{ccccc|ccccc|c}
 & & & & & & & & & & 0\\
 & & & & & & & & & & 0\\
 & & P(C_m) & & & & & -P(C_m) & & & \vdots\\
 & & & & & & & & & & 0\\
 & & & & & & & & & & 0\\
\hline
0 & 0 & \cdots & 0 & 0 & & & & & & 0\\
0 & 0 & \cdots & 0 & 0 & & & & & & 0\\
\vdots & \vdots & \ddots & \vdots & \vdots & & & P(C_m) & & & \vdots\\
0 & 0 & \cdots & 0 & 0 & & & & & & 0\\
0 & 0 & \cdots & 0 & 0 & & & & & & 0\\
\hline
0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 1
\end{array}\right]
\quad
W(Q_{2m})=\left[\begin{array}{ccccc|ccccc|c}
0 & 0 & \cdots & 0 & 0 & & & & & 0 & 0\\
0 & 0 & \cdots & 0 & 0 & & & & & 0 & 0\\
\vdots & \vdots & \ddots & \vdots & \vdots & & & I_k & & & \vdots\\
0 & 0 & \cdots & 0 & 0 & & & & & 0 & 0\\
0 & 0 & \cdots & 0 & 0 & -1 & -1 & -1 & -1 & 0 & 1\\
\hline
 & & & & 0 & 0 & 0 & \cdots & 0 & 0 & 0\\
 & & & & 0 & 0 & 0 & \cdots & 0 & 0 & 0\\
 & & I_k & & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots\\
 & & & & 0 & 0 & 0 & \cdots & 0 & 0 & 0\\
0 & 0 & \cdots & 0 & 0 & 1 & 1 & \cdots & 1 & 1 & -1\\
\hline
0 & 0 & \cdots & 0 & 1 & 0 & 0 & \cdots & 0 & 0 & 0
\end{array}\right]$$

Where k= $(\alpha_1+1).(\alpha_2+1)\ldots.(\alpha_n+1)-1$.they are $[2(\alpha_1+1).(\alpha_2+1)\ldots.(\alpha_n+1)+1]\times[2(\alpha_1+1).(\alpha_2+1)\ldots.(\alpha_n+1)+1]$ square matrix.

III. THE MAIN RESULTS

In this section we find the general form of The Rational characters, The Artin's characters table and The Artin Cokernel of the group $(Q_{2m}\times C_4)$ when m is an odd number .

Proposition 6: The rational valued characters table of the group $(Q_{2m}\times C_4)$ when m is an odd number is equal to the tensor product of the rational valued characters table of Q_{2m} when m is an odd number and the rational valued characters table of C_4 that is: $\overset{*}{\equiv}(Q_{2m}\times C_4)=\overset{*}{\equiv}(Q_{2m})\otimes\overset{*}{\equiv}(C_4)$.

Proof :-

Table 2 The rational characters table of C_4

$C_4=\{I, c, c^2, c^3\}$

Since $\overset{*}{\equiv}(C_4)=$

	h_1'	h_2'	h_3'
χ_1'	2	-2	**0**
χ_2'	1	1	**-1**
χ_3'	**1**	**1**	**1**

$h_1'=\{(I)\}$, $h_2'=\{c^2\}$, $h_3'=\{c,c^3\}$ then,

$\chi_1'(h_1')=\theta_1'(h_1')=2$

$\chi_1'(h_2')=\theta_1'(h_2')=-2$

$\chi_1'(h_3')=\theta_1'(h_3')=0$

$\chi_2'(h_1')=\chi_2'(h_2')=\theta_2'(h_1')=\theta_2'(h_2')=1$

$\chi_2'(h_3')=\theta_2'(h_3')=-1$

$\chi_3'(h_1')=\chi_3'(h_2')=\chi_3'(h_3')=\theta_3'(h_1')=\theta_3'(h_2')=\theta_3'(h_3')=1$

From the definition of $Q_{2m}\times C_4$,

and Theorem(1.3) we have

$\equiv(Q_{2m}\times C_4)=(\equiv Q_{2m})\otimes(\equiv C_4)$

Each element in $Q_{2m}\times C_4$

$h_{ng}=h_n\cdot h_g'$ $\forall h_n\in Q_{2m}$, $h_g'\in C_4$,

$n=1,2,3,\ldots,4m$, $\mathbf{g}\in\{I,c,c^2,c^3\}$,

each irreducible character of $Q_{2m}\times C_4$ is

$\chi_{(i,j)}=\chi_i.\chi_j'$ where χ_i is an irreducible character of Q_{2m}

and χ_j' is the irreducible character of C_4 ,then

$$\chi_{(i,j)}(h_{ng})=\begin{cases}2\chi_i(h_n) & if \quad j=1 \quad and \quad g\in\{I\}\\ -2\chi_i(h_n) & if \quad j=1 \quad and \quad g\in\{c^2\}\\ 0\chi_i(h_n) & if \quad j=1 \quad and \quad g\in\{c,c^3\}\\ \chi_i(h_n) & if \quad j=2 \quad and \quad g\in\{I,c^2\}\\ -\chi_i(h_n) & if \quad j=2 \quad and \quad g\in\{c,c^3\}\\ \chi_i(h_n) & if \quad j=3 \quad and \quad g\in C_4\end{cases}$$

From Corollary (1.2)

$\theta_{(i,j)} = \sum_{\sigma\in Gal(Q(\chi_{(i,j)})/Q)} \sigma(\chi_{(i,j)})$ where $\theta_{(i,j)}$ is the rational valued character of $Q_{2m}\times C_4$ Then,

$$\theta_{(i,j)}(h_{ng}) = \sum_{\sigma\in Gal(Q(\chi_{(i,j)}(h_{ng}))/Q)} \sigma(\chi_{(i,j)}(h_{ng}))$$

(I) (a) If j=1 and $g\in\{I\}$

$$\theta_{(i,j)}(h_{ng}) = \theta_{(i,j)}(h_{ng}) = \sum_{\sigma\in Gal(Q(\chi_i(h_n))/Q)} \sigma(2\chi_i(h_n)) = 2\sum_{\sigma\in Gal(Q(\chi_i(h_n))/Q)} \sigma(\chi_i(h_n)) = \theta_i(h_n)\,.\,2$$

$= \theta_i(h_n)\,.\,\theta'_j(h'_g)$

(b) If j=1 and $g\in\{c^2\}$

$$\theta_{(i,j)}(h_{ng}) = \sum_{\sigma\in Gal(Q(\chi_I(h_n))/Q)} \sigma(-2\chi_i(h_n)) = -2\sum_{\sigma\in Gal(Q(\chi_i(h_n))/Q)} \sigma(\chi_i(h_n))$$

$= \theta_i(h_n)\,.\,-2 = \theta_i(h_n)\,.\,\theta'_j(h'_g)$

(c) if j=1 and $g\in\{c,^3\}$

$$\theta_{(i,j)}(h_{ng}) = \sum_{\sigma\in Gal(Q(\chi_I(h_n))/Q)} \sigma(0\chi_i(h_n)) = 0 \sum_{\sigma\in Gal(Q(\chi_I(h_n))/Q)} \sigma(\chi_i(h_n)) = \theta_i(h_n)\,.\,0 = \theta_i(h_n)\,.\,\theta'_j(h'_g)$$

where θ_i is the rational valued character of Q_{2m} .

(II) (a) If j=2 and $g\in\{I,c^2\}$

$$\theta_{(i,j)}(h_{ng}) = \sum_{\sigma\in Gal(Q(\chi_i(h_n))/Q)} \sigma(\chi_i(h_n)) = \theta_i(h_n)\,.\,1 = \theta_i(h_n)\,.\,\theta'_j(h'_g)$$

(b) If j=2 and $g\in\{c,c^3\}$

$$\theta_{(i,j)}(h_{ng}) = \sum_{\sigma\in Gal(Q(\chi_i(h_n))/Q)} \sigma(-\chi_i(h_n)) = -\sum_{\sigma\in Gal(Q(\chi_i(h_n))/Q)} \sigma(\chi_i(h_n))$$

$$= \sum_{\sigma\in Gal(Q(\chi_i(h_n))/Q)} \sigma(\chi_i(h_n))\,.\,-1 = \theta_i(h_n)\,.-1 =$$

$\theta_i(h_n)\,.\,\theta'_j(h'_g)$.

(III) If j=3 and $g\in C_4$

$$\theta_{(i,j)}(h_{ng}) = \sum_{\sigma\in Gal(Q(\chi_i(h_n))/Q)} \sigma(\chi_i(h_n)) = \theta_i(h_n)\,.\,1 = \theta_i(h_n)\,.\,\theta'_j(h'_g)$$

From [I], [II] and [III] we have

$\theta_{(i,j)} = \theta_i\,.\,\theta'_j$.

Then $\equiv^*(Q_{2m}\times C_4) = \equiv^*(Q_{2m}) \otimes \equiv^*(C_4)$.

Example 1: To find $\equiv^*(Q_{10}\times C_4)$by using the Proposition 6 we get the following table:

$$\begin{bmatrix}
8 & -2 & 8 & -2 & 0 & -8 & 2 & -8 & 2 & 0 & 0 & 0 & 0 & 0 & 0\\
2 & 2 & 2 & 2 & 2 & -2 & -2 & -2 & -2 & -2 & 0 & 0 & 0 & 0 & 0\\
8 & -2 & -8 & 2 & 0 & -8 & 2 & 8 & -2 & 0 & 0 & 0 & 0 & 0 & 0\\
2 & 2 & 2 & 2 & -2 & -2 & -2 & -2 & -2 & 2 & 0 & 0 & 0 & 0 & 0\\
4 & 4 & -4 & -4 & 0 & -4 & -4 & 4 & 4 & 0 & 0 & 0 & 0 & 0 & 0\\
4 & -1 & 4 & -1 & 0 & 4 & -1 & 4 & -1 & 0 & -4 & 1 & -4 & 1 & 0\\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1\\
4 & -1 & -4 & 1 & 0 & 4 & -1 & -4 & 1 & 0 & -4 & 1 & 4 & -1 & 0\\
1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & 1\\
2 & 2 & -2 & -2 & 0 & 2 & 2 & -2 & -2 & 0 & -2 & -2 & 2 & 2 & 0\\
4 & -1 & 4 & -1 & 0 & 4 & -1 & 4 & -1 & 0 & 4 & -1 & 4 & -1 & 0\\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1\\
4 & -1 & -4 & 1 & 0 & 4 & -1 & -4 & 1 & 0 & 4 & -1 & -4 & 1 & 0\\
1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 & 1 & -1\\
2 & 2 & -2 & -2 & 0 & 2 & 2 & -2 & -2 & 0 & 2 & 2 & -2 & -2 & 0
\end{bmatrix}$$

Proposition 7: The If m= $p_1^{\alpha_1}.p_2^{\alpha_2}.......p_n^{\alpha_n}$ where g.c.d(p_i,p_j)=1 , if i≠j and p_i 's are primes numbers, and α_n any positive integers, then the Artin's characters table of the quaternion group ($Q_{2m}\times C_4$) is:

Table 3 Rational characters table of the Quaternion group Q_{2m} when $m=2^h, h \in Z^+$

	Γ- classes of $(Q_{2m})\times\{I\}$				Γ- classes of $(Q_{2m})\times\{z^2\}$					Γ- classes of $(Q_{2m})\times\{z\}$				
Γ-classes	$[1,I]$	$[x^m,I]$	..	$[x,I]$	$[y,I]$	$[I,z^2]$	$[x^m,z^2]$	…	$[x,z^2]$	$[y,z^2]$	$[I,z]$	$[x^m,z]$	…	$[x,z]$	$[y,z]$
$\lvert CL_\alpha \rvert$	**1**	**1**	..	**2**	**m**	**1**	**1**	·	**2**	**m**	**1**	**1**	…	**2**	**m**
$\lvert C_{Q_{2m}\times C_4}(CL_\alpha)\rvert$	**16m**	**16m**	..	**8m**	**16**	**16m**	**16m**	·	**8m**	**16**	**16m**	**16m**	…	**8m**	**16**
$\Phi_{(1,1)}$	**4Ar(Q_{2m})**					**0**					**0**				
$\Phi_{(2,1)}$															
⋮															
$\Phi_{(l,1)}$															
$\Phi_{(l+1,1)}$															
$\Phi_{(1,2)}$	**2Ar(Q_{2m})**					**2Ar(Q_{2m})**					**0**				
$\Phi_{(2,2)}$															
⋮															
$\Phi_{(l,2)}$															
$\Phi_{(l+1,2)}$															
$\Phi_{(1,3)}$	**Ar(Q_{2m})**					**Ar(Q_{2m})**					**Ar(Q_{2m})**				
$\Phi_{(2,3)}$															
⋮															
$\Phi_{(l,3)}$															

$\Phi_{(l+1,3)}$			

which is $[6(r_1+1)(r_2+1)\cdots(r_n+1)+3]\times[6(r_1+1)(r_2+1)\cdots(r_n+1)+3]$ square matrix

Proof : Let $g \in (Q_{2m}\times C_4)$;g=(q,I) or g=(q,z) or $g=(q,z^2)$or $g=(q,z^3)$ $q \in Q_{2m}$,$I,z,z^2,z^3 \in C_4$

Case (I):

If H is a cyclic subgroup of $Q_{2m} \times \{I\}$,then:

1.$H=\langle(x, I)\rangle$ 2. $H=\langle(y, I)\rangle$

And φ the principal character of H, Φ_j Artin characters of Q_{2m} where $1 \le j \le l+2$ then by using Theorem 2

1. $H=\langle(x, I)\rangle$

(i) If g=(1,I) and g∈H

$$\Phi_{(j,1)}((1,I)) = \frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot\varphi(g) = \frac{16m}{|C_H(I,1)|}.1 = \frac{4.4m}{|C_H(I,1)|}.1 = \frac{4|C_{Q_{2m}}(1)|}{|C_{\langle x\rangle}(1)|}.\varphi(1) = 4.\Phi_j(1)$$

since H ∩CL(1,I)={(1,I)}

(ii) if g=(x^m, I) and g∈H

$$\Phi_{(j,1)}(g) = \frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot\varphi(g)$$

$$= \frac{16m}{|C_H(g)|}.1 = \frac{4.4m}{|C_H(g)|}.1 = \frac{4|C_{Q_{2m}}(x^m)|}{|C_{\langle x\rangle}(x^m)|}.\varphi(g) = 4.\Phi_j(x^m)$$

since H ∩CL(g)={g},φ(g)=1

(iii) if g= $(x^i, I), i \ne m$ and $i \ne 2m$ and g∈H

$$\Phi_{(j,1)}(g) = \frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot(\varphi(g)+\varphi(g^{-1})) = \frac{8m}{|C_H(g)|}(1+1) = \frac{4.2m}{|C_H(g)|}.(1+1) = \frac{4|C_{Q_{2m}}(q)|}{|C_{\langle x\rangle}(q)|}.(\varphi(g)+\varphi(g^{-1})) = 4.\Phi_j(q)$$

since H∩CL(g)={g,g^{-1}} and $\varphi(g)=\varphi(g^{-1})=1$,g=(q,I),q∈ Q_{2m} and $q \ne x^m$,q≠1.

(iv) if g ∉ H Since H∩CL(g)= ϕ

2. $H=\langle(y, I)\rangle$ ={(1,I),(y,I),(y^2,I),(y^3,I)}

(i) If g=(1,I) H∩CL(1,I)={(1,I)}

$$\Phi_{(l+1,1)}(g) = \frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot\varphi(g) = \frac{16m}{4}.1 = 4m = 4.\Phi_{l+1}(1)$$

(ii) If g= (x^m, I)=(y^2,I) and g∈ H

$$\Phi_{(l+1,1)}(g) = \frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot\varphi(g) = \frac{16m}{4}.1 = 4m = 4.\Phi_{l+1}(x^m)$$

Since H∩CL(g)={g} ,φ(g)=1

(iii) g=(y,I) or g=(y^3,I) and g∈H

$$\Phi_{(l+1,1)}(g)=\frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot(\varphi(g)+\varphi(g^{-1}))=\frac{16}{4}.(1+1)=4.2=4.\Phi_{l+1}(y)$$

since H∩CL(g)={g,g^{-1}} and φ(g)=φ(g^{-1})=1

Otherwise

$$\Phi_{(l+1,1)}(g)=0 \quad \text{since } H\cap CL(g)=\emptyset$$

Case (II):

If H is a cyclic subgroup of $Q_{2m}\times\{z^2\}$,then:

1.H=⟨(x, I)⟩=$\langle(x,z^2)\rangle$ 2. H=⟨(y, I) = $\langle(y,z^2)\rangle$⟩

And φ the principal character of H, Φ_j Artin characters of Q_{2m} where $1\le j\le l+2$ then by using Theorem 2

1. H=⟨(x, I)⟩ = $\langle(x,z^2)\rangle$

(i) If g=(1,I) or g=(1,z^2) and g∈H

$$\Phi_{(j,2)}((1,I))=\frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot\varphi(g)=\frac{16m}{|C_H(I,1)|}.1=\frac{4.4m}{|C_H(I,1)|}.1=\frac{4|C_{Q_{2m}}(1)|}{2|C_{\langle x\rangle}(1)|}.\varphi(1)=2.\Phi_j(1)$$

since H∩CL(1,I)={(1,I),(1,z^2)}

(ii) if g=(x^m,I) and g∈ H

$$\Phi_{(j,2)}(g)=\frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot\varphi(g)=\frac{16m}{|C_H(g)|}.1=\frac{4.4m}{|C_H(g)|}.1=\frac{4|C_{Q_{2m}}(x^m)|}{2|C_{\langle x\rangle}(x^m)|}.\varphi(g)=2.\Phi_j(x^m)$$

since H ∩CL(g)={g},φ(g)=1

(iii) if g=$(x^i,I), i\ne m$ and $i\ne 2m$ and g∈H

$$\Phi_{(j,2)}(g)=\frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot(\varphi(g)+\varphi(g^{-1}))=\frac{8m}{|C_H(g)|}(1+1)=\frac{4.2m}{|C_H(g)|}.(1+1)=\frac{4|C_{Q_{2m}}(q)|}{2|C_{\langle x\rangle}(q)|}.(\varphi(g)+\varphi(g^{-1}))=2.\Phi_j(q)$$

since H∩CL(g)={g,g^{-1}} and φ(g)=φ(g^{-1})=1,g=(q,I),q∈ Q_{2m}

and $q\ne x^m, q\ne 1$

(iv) if g∉H Since H∩CL(g)= ϕ

2. H=⟨(y, I)⟩ ={(1,I),(y,I),(y^2,I),(y^3,I), (1, z^2),(y, z^2),(y^2, z^2),(y^3,z^2)}

(i) If g=(1,I) or g=(1,z^2) H∩CL(1,I)={(1,I), (1,z^2)}

$$\Phi_{(l+1,2)}(g)=\frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot\varphi(g)=\frac{16m}{8}.1=2m=2.\Phi_{l+1}(1)$$

$$\Phi_{(j,2)}(g)=2.0=2.\Phi_j(q)$$

(ii) If g= (x^m,I)=(y^2,I) or g=(y^2,z^2) and g∈ H

$$\Phi_{(l+1,2)}(g)=\frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot\varphi(g)=\frac{16m}{8}.1=2m=2.\Phi_{l+1}(x^m)$$

Since H∩CL(g)={g} ,φ(g)=1

(iii) g=(y,I) or g=(y^3,I) or g=(y,z^2) or g=(y^3, z^2) and g∈H

$$\Phi_{(l+1,2)}(g)=\frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot(\varphi(g)+\varphi(g^{-1}))=\frac{16}{8}.(1+1)=2.2=2.\Phi_{l+1}(y)$$

since H∩CL(g)={g,g^{-1}} and φ(g)=φ(g^{-1})=1

Otherwise

$$\Phi_{(l+1,2)}(g)=0 \qquad \text{since } H\cap CL(g)=$$

<u>Case (III):</u>

If H is a cyclic subgroup of (Q_{2m}× {z}),then:

1.H=$\langle(x,z)\rangle=\langle(x,z^2)\rangle=\langle(x,z^3)\rangle$

2. H=$\langle(y,z)\rangle=\langle(y,z^2)\rangle=\langle(y,z^3)\rangle$

And φ the principal character of H, Φ_j Artin characters of Q_{2m} where $1\le j\le l+2$ then by using Theorem 2

1.*H=*$\langle(x,z)\rangle$

(i) If g=(1,I) or g=(1,z)or g=(1,z^2) or g=(1,z^3) and g∈H

$$\Phi_{(j,3)}(g)=\frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(1,I)|}\cdot\varphi(g)=\frac{16m}{|C_H(1,I)|}.1=\frac{4.4m}{|C_{\langle(x,z)\rangle}(1,I)|}.1=\frac{4|C_{Q_{2m}}(1)|}{4|C_{\langle x\rangle}(1)|}.\varphi(1)=\Phi_j(1)$$

since H ∩CL(g)={(1,I),(1,z),(1,z^2),(1,z^3)}

(ii) If g=(1,I) or g=(x^m,I) or g=(x^m,z) or g=(1,z)or g=(x^m,z^2) or g=(1,z^2)or g=(1,z^3) or g=(x^m,z^3) and g∈H

(a) if g=(1,I) or g=(1,z) or g=(1,z^2) or g=(1,z^3) and g∈H.

$$\Phi_{(j,3)}(g)=\frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot\varphi(g)=\frac{16m}{|C_H(g)|}.1=\frac{4.4m}{|C_{\langle(x,z)\rangle}(g)|}.1=\frac{4|C_{Q_{2m}}(1)|}{4|C_{\langle x\rangle}(1)|}.\varphi(1)=\Phi_j(1)$$

since H ∩CL(g)={g},φ(g)=1

(b)If g=(x^m,I) or g=(x^m,z) or g=(x^m,z^2) or g=(x^m,z^3) and g∈H

$$\Phi_{(j,3)}(g)=\frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot\varphi(g)=\frac{16m}{|C_H(g)|}.1=\frac{4.4m}{|C_H(g)|}.1=\frac{4|C_{Q_{2m}}(x^m)|}{4|C_{\langle x\rangle}(x^m)|}.\varphi(x^m)=\Phi_j(x^m)$$

since H ∩CL(g)={g},φ(g)=1

(iii) If $g=\{(x^i,I),(x^i,z),(x^i,z^2),(x^i,z^3)\}$,*i≠m,i≠2m* and g∈H

$$\Phi_{(j,3)}(g)=\frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot(\varphi(g)+\varphi(g^{-1}))=\frac{8m}{|C_H(g)|}(1+1)\ \frac{4.2m}{|C_H(g)|}.(1+1)=\frac{4|C_{Q_{2m}}(q)|}{4|C_{\langle x\rangle}(q)|}.(\varphi(g)+\varphi(g^{-1}))=\Phi_j(q)$$

since H∩CL(g)={g,g^{-1}} and φ(g)=φ(g^{-1})=1,g=(q,z)=(q,z^3),q∈ Q_{2m} and q≠ x^m ,q≠1

(i) if g∉H Since H∩CL(g)= ϕ

$$\Phi_{(j,)}(g)=0$$

2. H= $\langle (y,z) \rangle$ ={(1,I),(y,I),(y^2,I),(y^3,I),(1,z),(y,z),(y^2,z),(y^3,z), (1,z^2),(y,z^2),(y^2,z^2),(y^3,z^2),(1,z^3),(y,z^3),(y^2,z^3),(y^3,z^3)}

(i) If g=(1,I) or g=(1,z) or g=(1,z^2) or g=(1,z^3)and g∈H H∩CL(g)={(1,I),(1,z),(1,z^2) ,(1,z^3)}

$$\Phi_{(l+1,3)}(g) = \frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot\varphi(g) = \frac{16m}{16}.1 = m = .\Phi_{l+1}(1)$$

(ii) If g= (x^m, I)=(y^2,I) or g= (y^2,z) or g=(y^2,z^2) or g=(y^2,z^3) and g∈ H

$$\Phi_{(l+1,3)}(g) = \frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot\varphi(g) = \frac{16m}{16}.1 = m = \Phi_{l+1}(x^m)$$

Since H∩CL(g)={g} ,φ(g)=1

(iii) g=(y,I)or g= (y,z)or g=(y,z^2) or g=(y,z^3) or g=(y^3,I_2)or g= (y^3,z) or g= (y^3,z^2) or g= (y^3,z^3) and g∈H

$$\Phi_{(l+1,3)}(g) = \frac{|C_{Q_{2m}\times C_4}(g)|}{|C_H(g)|}\cdot(\varphi(g)+\varphi(g^{-1})) = \frac{16}{16}.(1+1) = 2 = \Phi_{l+1}(y)$$

since H∩CL(g)={g,g^{-1}} and φ(g)=φ(g^{-1})=1

Otherwise

$\Phi_{(l+1,3)}(g) = 0$ since H∩CL(g)=∅

Example 2: To find Ar ($Q_{10}\times C_4$)by using the Proposition 7 we get the following table

$$\begin{bmatrix}
40 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
20 & 20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
8 & 0 & 8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
4 & 4 & 4 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
20 & 0 & 20 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
20 & 0 & 0 & 0 & 0 & 20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
10 & 10 & 0 & 0 & 0 & 10 & 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
4 & 0 & 4 & 0 & 0 & 4 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
2 & 2 & 2 & 2 & 0 & 2 & 2 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 0\\
10 & 0 & 10 & 0 & 2 & 10 & 0 & 10 & 0 & 2 & 0 & 0 & 0 & 0 & 0\\
10 & 0 & 0 & 0 & 0 & 10 & 0 & 0 & 0 & 0 & 10 & 0 & 0 & 0 & 0\\
5 & 5 & 0 & 0 & 0 & 5 & 5 & 0 & 0 & 0 & 5 & 5 & 0 & 0 & 0\\
2 & 0 & 2 & 0 & 0 & 2 & 0 & 2 & 0 & 0 & 2 & 0 & 2 & 0 & 0\\
1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 0\\
5 & 0 & 5 & 0 & 1 & 5 & 0 & 5 & 0 & 1 & 5 & 0 & 5 & 0 & 1
\end{bmatrix}$$

Proposition 8: If m= $p_1^{\alpha_1}.p_2^{\alpha_2}.......p_n^{\alpha_n}$ where g.c.d(p_i,p_j)=1 , if i≠j and p_i 's are primes numbers, and α_n any positive integers, then the matrix M($Q_{2m}\times C_4$) of the quaternion group ($Q_{2m}\times C_4$) is:

$$M(Q_{2m}\times C_4) = \left[\begin{array}{c|c|c} M(Q_{2m}) & M(Q_{2m}) & M(Q_{2m}) \\ \hline 0 & M(Q_{2m}) & M(Q_{2m}) \\ \hline 0 & 0 & M(Q_{2m}) \end{array}\right]$$

which is [6(r_1+1)(r_2+1)⋯(r_n+1)+3]×[6(r_1+1)(r_2+1)⋯(r_n+1)+3] square matrix M (Q_{2m}) is similar to the matrix of the proposition 4 .

Proof : By Proposition 7 we obtain the Artin's characters Table $Ar(Q_{2m}\times C_4)$ of the group $(Q_{2m}\times C_4)$ when m= $p_1^{\alpha_1}.p_2^{\alpha_2}.......p_n^{\alpha_n}$ $\alpha_n \in Z^+$ and p_i is prime number and from the Proposition 6 we get the rational valued characters table $(\overset{*}{\equiv}(Q_{2m}\times C_4))$ of the group $(Q_{2m}\times C_4)$ when m= $p_1^{\alpha_1}.p_2^{\alpha_2}.......p_n^{\alpha_n}$ $,\alpha_n \in Z^+$ and p_i is prime number.

Thus, by definition of M(G) we can find the matrix $M(Q_{2m}\times C_4)$ when $p_1^{\alpha_1}.p_2^{\alpha_2}.......p_n^{\alpha_n}$ $,\alpha_n \in Z^+$ and p_i is prime number.

$$M(Q_{2m}\times C_4) = Ar(Q_{2m}\times C_4)\cdot(\overset{*}{\equiv}(Q_{2m}\times C_4))^{-1} = \left[\begin{array}{c|c|c} M(Q_{2m}) & M(Q_{2m}) & M(Q_{2m}) \\ \hline 0 & M(Q_{2m}) & M(Q_{2m}) \\ \hline 0 & 0 & M(Q_{2m}) \end{array}\right] = M(Q_{2m}\times C_4)$$

Example 3: Consider the group $(Q_{10}\times C_4)$, we can find the matrix $M(Q_{10}\times C_4)$ by using:

$$M(Q_{10}\times C_4) = Ar(Q_{10}\times C_4)\cdot(\overset{*}{\equiv}Q_{10}\times C_4)^{-1} = \begin{bmatrix} 2&1&2&1&1&2&1&2&1&1&2&1&2&1&1 \\ 0&1&0&1&1&0&1&0&1&1&0&1&0&1&1 \\ 0&1&0&1&0&0&1&0&1&0&0&1&0&1&0 \\ 0&1&0&1&0&0&1&0&1&0&0&1&0&1&0 \\ 1&1&0&0&0&1&1&0&0&0&1&1&0&0&0 \\ 0&0&0&0&0&2&1&2&1&1&2&1&2&1&1 \\ 0&0&0&0&0&0&1&0&1&1&0&1&0&1&0 \\ 0&0&0&0&0&0&1&0&1&0&0&1&0&1&0 \\ 0&0&0&0&0&0&1&0&1&0&0&1&0&1&0 \\ 0&0&0&0&0&1&1&0&0&0&1&1&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&2&1&2&1&1 \\ 0&0&0&0&0&0&0&0&0&0&0&1&0&1&0 \\ 0&0&0&0&0&0&0&0&0&0&0&1&0&1&0 \\ 0&0&0&0&0&0&0&0&0&0&0&1&0&1&0 \\ 0&0&0&0&0&0&0&0&0&0&1&1&0&0&0 \end{bmatrix}$$

Proposition 9: If m= $p_1^{\alpha_1}.p_2^{\alpha_2}.......p_n^{\alpha_n}$ where $g.c.d(p_i,p_j)=1$, if $i\neq j$ and p_i 's are primes numbers, and α_n any positive integers, then the matrices $P(Q_{2m}\times C_4)$ and $W(Q_{2m}\times C_4)$ are taking the forms :

$$P(Q_{2m}\times C_4) = \left[\begin{array}{c|c|c} 0 & 0 & P(Q_{2m}) \\ \hline 0 & P(Q_{2m}) & -P(Q_{2m}) \\ \hline P(Q_{2m}) & -P(Q_{2m}) & 0 \end{array}\right]$$

Which is $[6(r_1+1)(r_2+1)\cdots(r_n+1)+3]\times[6(r_1+1)(r_2+1)\cdots(r_n+1)+3]$ square matrix .

And

$$W(Q_{2m}\times C_4) = \left[\begin{array}{c|c|c} W(Q_{2m}) & 0 & 0 \\ \hline 0 & W(Q_{2m}) & 0 \\ \hline 0 & 0 & W(Q_{2m}) \end{array}\right]$$

which is $[6(r_1+1)(r_2+1)\cdots(r_n+1)+3]\times[6(r_1+1)(r_2+1)\cdots(r_n+1)+3]$ square matrix .

<u>Proof :</u>

By using the proposition 8 taking the matrix $M(Q_{2m}\times C_4)$ and the above forms $P(Q_{2m}\times C_4)$ and $W(Q_{2m}\times C_4)$ then we have : $P(Q_{2m}\times C_4).\ M(Q_{2m}\times C_4).\ W(Q_{2m}\times C_4)$ =

$$\text{diag}\ \{\underbrace{2,2,2,2,\cdots,2}_{[6(r_1+1)(r_2+1)\cdots(r_n+1)-6]-time},1,1,1,1,1,1,1,1,1\}$$

$$=D(Q_{2m}\times C_4)$$

which is $[6(r_1+1)(r_2+1)\cdots(r_n+1)+3]\times[6(r_1+1)(r_2+1)\cdots(r_n+1)+3]$ square matrix .

Example 4: To find the matrices $P(Q_{10}\times C_4)$ and $W(Q_{10}\times C_4)$ by the proposition 5 to find $P(Q_{10})$ and $W(Q_{10})$:

$$P(Q_{10}\times C_4)=\begin{bmatrix}
0&0&0&0&0&0&0&0&0&0&1&-1&-1&1&0\\
0&0&0&0&0&0&0&0&0&0&0&1&0&-1&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&-1&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&1\\
0&0&0&0&0&1&-1&-1&1&0&-1&1&1&-1&0\\
0&0&0&0&0&0&1&0&-1&0&0&-1&0&1&0\\
0&0&0&0&0&0&0&1&-1&0&0&0&-1&1&0\\
0&0&0&0&0&0&0&0&1&0&0&0&0&-1&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&-1\\
1&-1&-1&1&0&-1&1&1&-1&0&0&0&0&0&0\\
0&1&0&-1&0&0&-1&0&1&0&0&0&0&0&0\\
0&0&1&-1&0&0&0&-1&1&0&0&0&0&0&0\\
0&0&0&1&0&0&0&0&-1&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&-1&0&0&0&0&0
\end{bmatrix}\quad and\quad W(Q_{10}\times C_4)=\begin{bmatrix}
0&0&1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&-1&0&1&0&0&0&0&0&0&0&0&0&0\\
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&1&1&-1&0&0&0&0&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&-1&0&1&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&1&1&-1&0&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&-1&0&1\\
0&0&0&0&0&0&0&0&0&0&1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&1&-1\\
0&0&0&0&0&0&0&0&0&0&0&1&0&0&0
\end{bmatrix}$$

And by the proposition 9 then:

$$P(Q_{10}\times C_4)=\left[\begin{array}{c|c|c} 0 & 0 & P(Q_{10}) \\ \hline 0 & P(Q_{10}) & -P(Q_{10}) \\ \hline P(Q_{10}) & -P(Q_{10}) & 0 \end{array}\right] \quad and \quad W(Q_{10}\times C_4)=\left[\begin{array}{c|c|c} W(Q_{10}) & 0 & 0 \\ \hline 0 & W(Q_{10}) & 0 \\ \hline 0 & 0 & W(Q_{10}) \end{array}\right]$$

Example 5: To find $D(Q_{10}\times C_4)$ and the cyclic decomposition of the factor group

We find the matrices $P(Q_{10}\times C_4)$ and $W(Q_{10}\times C_4)$ as in example 4 and $M(Q_{10}\times C_4)$ as in example 3,then :

$P(Q_{10}\times C_4).M(Q_{10}\times C_4).W(Q_{10}\times C_4)=$

$\mathrm{diag}\{2,2,2,2,2,2,1,1,1,1,1,1,1,1,1\}=D(Q_{10}\times C_4)$

Then by Theorem 6 we have

$$AC(D(Q_{10}\times C_4)= \bigoplus_{i=1}^{6} C_2$$

The following theorem gives the cyclic decomposition of the factor group $AC(D(Q_{2m}\times C_4))$ when m= $p_1^{\alpha_1}.p_2^{\alpha_2}.......p_n^{\alpha_n}$ where g.c.d(p_i,p_j)=1 , if i≠j and p_i 's are primes numbers, and α_n any positive integers, .

Theorem 6: If m= $p_1^{\alpha_1}.p_2^{\alpha_2}.......p_n^{\alpha_n}$ where g.c.d(p_i,p_j)=1 , if i≠j and p_i 's are primes numbers, and α_n any positive integers, then the cyclic decomposition of $AC(Q_{2m}\times C_4)$ is :

$$AC(D(Q_{2m}\times C_4))= \bigoplus_{i=1}^{6(r_1+1)(r_2+1)\cdots(r_n+1)-6} C_2$$

Proof : By using the proposition 8,we can find matrix $M(Q_{2m}\times C_4)$ and by the proposition 9,we find $P(Q_{2m}\times C_4)$ and $W(Q_{2m}\times C_4)$ when $p_1^{\alpha_1}.p_2^{\alpha_2}.......p_n^{\alpha_n}$ where g.c.d(p_i,p_j)=1 , if i≠j and p_i 's are primes numbers, and α_n any positive integers:

$P(Q_{2m}\times C_4).\ M(Q_{2m}\times C_4).\ W(Q_{2m}\times C_4)=$

$\mathrm{diag}\{2,2,2,2,2,2,\ldots,2,2,2,1,1,1,1,1,1,1,1,1\}$

Then ,by the theorem 6 we have :

$$AC(D(Q_{2m}\times C_4))= \bigoplus_{i=1}^{6(r_1+1)(r_2+1)\cdots(r_n+1)-6} C_2$$

Example 6: Consider the groups ($Q_{17718750}\times C_4$) ,($Q_{12250}\times C_4$),then :

1. $AC(Q_{17718750} \times C_4) = AC(Q_{2.3^4.7.5^6} \times C_4) = \bigoplus_{i=1}^{414} C_2$

2. $AC(Q_{12250} \times C_4) = AC(Q_{2.7^2.5^3} \times C_4) = \bigoplus_{i=1}^{66} C_2$

REFERENCES

[1] A.H.Abdul-Munem, On Artin Cokernel of The Quaternion group Q_{2m} When m is an Odd number , M.Sc. thesis,University of Kufa, 2008.

[2] A.S.Abid, "Artin's Characters Table of Dihedral Group for Odd Number", M.Sc.thesis, University Kufa, 2006.

[3] C. Curits and I. Reiner, " Methods of Representation Theory with Application to Finite Groups and Order",John wily & sons, New York,1981.

[4] C.W. Curits & I. Renier " Representation Theory of Finite Groups and Associative Algebra " ,AMS Chelsea publishing ,1962 , printed by the AMS .

[5] H.R. Yassien, " On Artin Cokernel of Finite Group", M.Sc. thesis, Babylon University, 2000.

[6] I. M. Isaacs, "On Character Theory of Finite Groups ", Academic Press, New York, 1976.

[7] K. Sekigvchi," Extensions and the Irreducibilities of The Induced Characters of Cyclic p-Group", Hiroshima math Journal, p165-178, 2002.

[8] M. J. Hall, " The Theory of Group ", Macmillan, New York, 1959.

[9] T.Y. Lam," Artin Exponent of Finite Groups ", Columbia University, New York, 1967.